Scale invariance of equivalent utility principle under cumulative prospect theory

Jacek Chudziak, Marcin Halicki, Sebastian Wójcik

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$$\mathbf{E_g(X) = \int_{-\infty}^{0} (g(P(X > t)) - 1)\, dt + \int_{0}^{\infty} g(P(X > t))\, dt}.$$

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$$\mathbf{E_g(X)} = \int_{-\infty}^{0} (\mathbf{g}(\mathbf{P}(\mathbf{X} > \mathbf{t})) - \mathbf{1})\, \mathbf{dt} + \int_{0}^{\infty} \mathbf{g}(\mathbf{P}(\mathbf{X} > \mathbf{t}))\, \mathbf{dt}.$$

The Choquet integral has several interesting properties. In particular, it is additive for comonotonic risks, positively homogeneous and monotonic.

Furthermore, if $\mathbf{x} < \mathbf{y}$, $p \in [0, 1]$ and X is such that

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Tversky and Kahneman (1992), using the concept of the rank-dependent utility model, created the **Cumulative Prospect Theory**. They assumed that probabilities of gains and losses are distorted in a different way. Moreover they introduced the **generalized Choquet integral**

$$\mathbf{E_{gh}(X) = E_g(\max\{X, 0\}) - E_h(\max\{-X, 0\})}$$

related to the distortion functions g (for gains) and h (for losses) and applied it to describe the mathematical foundations of the Cumulative Prospect Theory.

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2. Results

In what follows, given an $x > 0$ and a $p \in [0, 1]$, by (x, p) we denote the random variable X such that $P(X = 0) = 1 - p$ and $P(X = x) = p$.

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Let $a_1, a_2 \in (0,\infty) \setminus \{1\}$ be such that $\frac{\ln \mathbf{a_1}}{\ln \mathbf{a_2}}$ is irrational. If (2) holds for $w = 0$ and H is a_i-invariant for $i = 1,2$ and every $X \in \mathcal{X}_2$, then there exist $\alpha, \beta, d > 0$ such that

$$\mathbf{u}(\mathbf{x}) = \begin{cases} \alpha x^d & \textit{for } x \geq 0, \\ -\beta(-x)^d & \textit{for } x < 0. \end{cases}$$

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Conversely, if (3) holds and u is of the form (4) with some $\alpha, \beta, d > 0$ then H is scale invariant.

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$$\mathbf{h(p) = 1 - g(1-p)} \quad \textit{for} \quad \mathbf{p \in (0,1)}$$

and there exist a $c > 0$ such that

$$\mathbf{u(x) = cx} \quad \textit{for} \quad \mathbf{x} \in \mathbb{R}.$$

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