

CONCLUSIONS

1. Real estate markets (REMs) are highly inertial dynamical systems among other economic systems.
2. Housing prices in local REMs increase to a similar degree with a delay with respect to dominating REM in Warsaw; fluctuations are due to exogenous factors (aggregate macroeconomic fluctuations).
3. Exogenous factors differentiate between time constants and price equilibrium levels.
4. Local REMs form a network of critically damped harmonic oscillators, coupled to external stimuli, and smaller, neighbouring REMs.

HOUSING PRICES FROM 2006 TO 2014

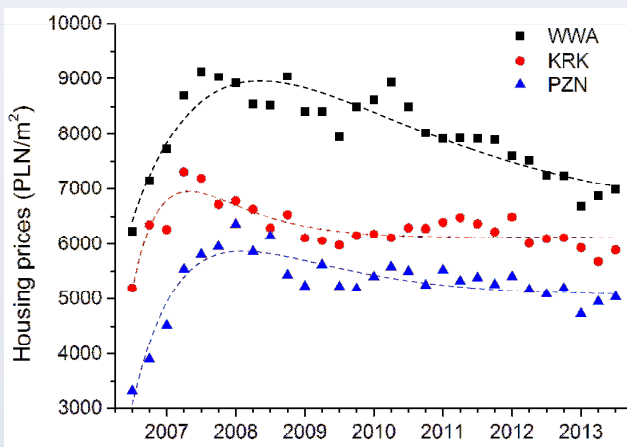


Fig. 1. Time-series of housing prices per square meter in: Warsaw (WWA), Cracow (KRK), and Poznan (PZN), from July 2006 to October 2013. Dashed lines show best fit obtained for overdamped harmonic oscillator.

EVOLUTION IN A PHASE SPACE

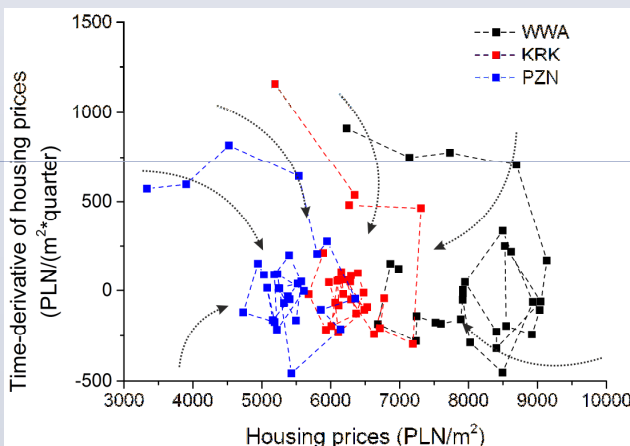


Fig. 2. Phase plots of the time-derivative of housing prices as a function of housing prices (per square meter) in: Warsaw (WWA), Cracow (KRK), and Poznan (PZN). Dashed lines represent flow directions.

CRITICALLY DAMPED HARMONIC OSCILLATOR

Forced harmonic oscillator is described by the equation:

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = F_{ext}$$

Total solution is composed of particular (x_p) and homogeneous (x_c) solutions:

$$x(t) = x_p + x_c = x_p + (At + B)e^{-\zeta t}$$

ESTIMATED PARAMETERS

	Time delay (quarters)	Decay constant (quarters)
WWA	0,0	7,4
KRK	1,0	2,5
PZN	1,6	4,2

Table 1. Time delays (with respect to WWA), and decay constants of critically damped oscillations obtained from non-linear fit procedure. Note that local real estate markets have large inertia due to large decay constants.

NETWORK OF COUPLED OSCILLATORS

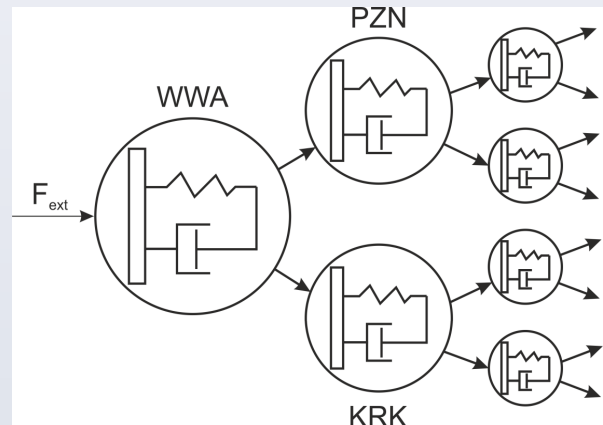


Fig. 3. Mechanical model of a network of local real estate markets represented by critically damped, harmonic oscillators forced by external stimuli, and further coupled with neighboring higher-order oscillators.

RECONSTRUCTED FLOW FIELD



Fig. 4. Hypothetical flow field reconstructed from the flow directions in the phase plot. In case of harmonic oscillator, such field corresponds to potential with quadratic curvature near its minima.

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