Scale invariance of equivalent utility principle under cumulative prospect theory

Jacek Chudziak
Faculty of Mathematics and Sciences, University of Rzeszów, ul. Prof. St. Pigonia 1, 35-310 Rzeszów, Poland
Marcin Halicki
Department of Regional Politics and Food Economy, University of Rzeszów, ul. Ćwiklińskiej 2, 35-601 Rzeszów, Poland
Sebastian Wójcik
Statistical Office in Rzeszów, ul. Jana III Sobieskiego 10, 35-959 Rzeszów, Poland

Consider an insurance company having the initial wealth w and a utility function u. The company covers a risk treated as a non-negative random variable. Roughly speaking, a premium principle is a rule for assigning a premium to an insurance risk. One of the frequently applied methods of pricing insurance contracts is the *Principle of Equivalent Utility* (cf. Gerber (1979)). Under the expected utility theory a premium principle $H(X)$ for risk X is a solution of the equation

$$u(w) = E[u(w + H(X) - X)]. \tag{1}$$

Equation (1) has the following interpretation: the utility of the initial wealth is equal to the expected utility of the surplus that results when a risk X is insured. A value of $H(X)$ is such that the insurer is indifferent between not accepting and accepting the insurance risk. Thus, this premium is called also the *indifference price of the insurer.* A solution $H(X)$ of (1) in the case $w = 0$ is called the *zero utility principle.*

In a recent paper Kałuszka and Krzeszowiec (2012) introduced a modification of the zero utility principle adjusted to the Cumulative Prospect Theory. This approach leads to the equation

$$u(w) = E_{gh}[u(w + H(X) - X)],$$

where $E_{gh}(X) = E_g(\max\{X, 0\}) - E_h(\max\{-X, 0\})$ is the generalized Choquet integral related to the distortion functions g (for gains) and h (for losses). In Kałuszka and Krzeszowiec (2012) several properties of the premium have been considered. One of them is a scale invariance, known also as a positive homogeneity. Let us recall that a premium principle is said to be *scale invariant* provided $H(aX) = aH(X)$ for all feasible risks X and $a > 0$. One can apply a no-arbitrage argument to justify this rule. Consider for instance the case $a = 2$. If the premium for $2X$ were greater than twice the premium of X, then one could buy insurance for $2X$ by buying two policies. Similarly, if the premium for $2X$ were less than twice the premium of X, then one could buy insurance for $2X$, sell insurance on X and X separately, and in this way make an arbitrage profit. Another justification for a scale invariance follows from the fact that a premium principle could be priced in two (or more) different currencies.

As it has been noted in Reich (1984), under expected utility theory, a scale invariance of a premium principle just for two particular values of parameter a implies its scale invariance. In this talk we extent that result onto the premium principles under Cumulative Prospect Theory. As a consequence of our results we obtain a generalization of Theorem 5 in Kałuszka and Krzeszowiec (2012).

References

[1] M. Kałuszka, M. Krzeszowiec, 2012, Pricing insurance contracts under Cumulative Prospect Theory, Insurance: Mathematics and Economics 50, 159-166.
[2] A. Reich, 1984, Homogeneous premium calculation principles, ASTIN Bulletin 14, 123-134.