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Nucleation, condensation and λ-transition within the complex network: An application to real-life market evolution

We fill a void in merging empirical and phenomenological characterisation of the dynamical phase transitions in complex systems by identifying three of them on real-life financial markets. We extract and interpret the empirical, numerical, and semi-analytical evidences for the existence of these phase transitions, by considering the Frankfurt Stock Exchange (FSE), as a typical example of a financial market of a medium size. Using the canonical object for the graph theory, i.e. the Minimal Spanning Tree (MST) network, we observe: (i) The initial phase transition from the equilibrium to non-equilibrium MST network in its nucleation phase, occurring at some critical time. Coalescence of edges on the FSE's transient leader is observed within the nucleation and is approximately characterized by the Lifsthiz-Slyozov growth exponent; (ii) The nucleation accelerates and transforms to the condensation process, in the second phase transition, forming a logarithmically diverging λ-peak of short-range order parameters at the subsequent critical time - an analogon of such a transition in superfluidity; (iii) In the third phase transition, the peak logarithmically decreases over three quarters of the year, resulting in a few loosely connected sub-graphs. This peak is reminiscent of a non-equilibrium superstar-like superhub or a `dragon king' effect, abruptly accelerating the evolution of the leader company. All these phase transitions are caused by the few richest vertices, which drift towards the leader and provide the most of the edges increasing the leader's degree. Thus, we capture an amazing phenomenon, likely of a more universal character, where a peripheral vertex becomes the one which is over dominating the complex network during an exceptionally long period of time.