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**Granger causality and transfer entropy for financial returns**

Causality is usually posed using two alternative scenarios:

i) the autoregressive, spectral based, causal Granger predictive modelling, and

ii) the information theoretic oriented, Kullback-Leibler (K-L) divergence based, Transfer Entropy formulation.

Both measures aspire to infer and quantify mutual directional causation, between coupled variables, or among multiple variables. The two formulations have been shown to be equivalent for Gaussian variables. Subsequent generalisation of this result has been obtained by Hlavackova-Schindler [Appl. Math. Sci. 73(5) (2011)].

Yet the precise equivalence and respective applicability validity criteria with surrogate analysis aiming at obtaining appropriate significance levels for the estimators are still lacking for the inference of causality using both approaches. This is particularly relevant in many complex phenomena where the observations and models are usually distributed following non-Gaussian statistics and possess both intrinsic intra-and inter-observable non-linear correlations.

As the Transfer Entropy formulation is widely used in applications to series of data from biomedical and similar analyses, it is natural that such studies address the question of generalization of Gaussianity to distributions characteristic for such data (e.g. in Hlavackova-Schindler: generalized normal, log-normal and multivariate Weinman exponential distribution).

But typical empirical distributions of returns from financial instruments are asymmetric, hence in econometric modeling skewed t-Student distribution or mixtures of Gaussian distributions, and other skewed distributions are used. In order to apply the TE method to testing interdependence between financial time series, we need to address the question of equivalency of GC and TE approaches for such empirical distributions. Estimates of applicability and confidence estimates for causation inference in the case of realistic data models will be reviewed for as far as they exist, and the possible directions for improvement will be discussed.