

Between diffusion and percolation: the fate of particles and people

S. R. Broadbent and J. M. Hammersley pointed out that in physical phenomena in which a fluid spreads randomly through a medium, according to nature of the problem one can ascribe the random mechanism either to the fluid or to the medium, [1]. The first alternative is a diffusion, while the second alternative is a percolation. In the simplest example of one-dimensional diffusion process a particle (of the fluid) takes steps of unit length along a straight line (integer number axis) and after any number of steps particle has, independently of its history, the same probability p of taking its next step to the right and probability $(1 - p)$ of taking its next step to the left. Then the position of the particle after n steps is given by the Bernoulli binomial formula. If $p=1/2$, then the variance equals n . This means that the variance growth linearly with the number of steps, [2].

In the analogous percolation process fluid and the medium are the same, but the randomisation mechanism resides in the medium, not in the particle. In an example of the percolation upon the number axis, each integer point of the axis has ascribed, with equal probability (each $1/2$) of being a right sense or a left sense point. The particle starts from the origin and takes steps of unit length, the direction of any step being that of the sense of the point from which that step starts. Thus the state of the medium completely determines the motion of the particle, which moves steadily in one direction until it encounters successive point of opposite sense, whereupon it oscillates between them. When n is infinite, the variance is of the order $n^{1/2}$. This means that there is probability 1 that the particle will visit only finite number of points.

Summing up, the percolation leads to trapping the particle in a well, from which it cannot run away, while in diffusion it is always free to move. In social analogy the percolation means an inevitably captivity and the diffusion means a freedom. The case of people travelling every day to work is an example. Their situation resembles this of a particle trapped in the hole in a porous medium. In social life this trap is not complete, every the worker has two free days, he has also a holidays. To describe such a situation more adequately, we propose as more realistic for description of the social situation, the mathematical model intermediary between the percolation and the diffusion, [3, 4]. .

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